Spectroscopic Study of Ion-Neutral Coupling in Plasma Acceleration

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The acceleration of a partially ionized gas by electromagnetic means might be quite inefficient, if the coupling between the ions and the neutrals, for momentum transfer, is weak. In this paper, the absence or presence of a sufficiently strong coupling is generally revealed by velocity disparities determined spectroscopically, for several species, both ionic and neutral. Velocities are determined by Doppler shift measurements of selected spectral lines. Results regarding two different situations are presented, from experiments where two basically similar axisymmetric configurations operate under significantly different conditions. In the first case, a large disparity in ion-neutral velocities is observed, whereas in the second case, the said disparity disappears and the common ion-neutral velocities are in good agreement with the center-of-mass velocity, as determined independently from measurements of motionally induced potentials in the flow. A satisfactory interpretation of the results is obtained when the calculated mean free path for momentum exchange is compared with a typical dimension of the electromagnetic acceleration region.

I. Introduction

CONSIDER a propulsion device operating at any desirable values of thrust and mass flow rate. It is well known that in such a device the thrust power (and therefore the input power) is affected by the state of uniformity of the propellant velocity. Specifically, spatial and/or temporal profiles of the propellant velocity have been considered in the literature as sources of nonuniformity losses. Strictly speaking, the mentioned losses are not associated with any kind of power dissipation. Actually, they reflect the fact that the power required for a given thrust, at a given flow rate, becomes increasingly larger, when the velocity nonuniformities become stronger.

Quantitative estimates of nonuniformity effects may be obtained from the following simple analysis. Consider two cases:

Case I

A mass flow rate $\dot{m}$ is uniformly accelerated to a velocity u, producing a thrust $F = \dot{m}u$, at the expense of a thrust power $P = F^2/2\dot{m}$.

Case II

In this case, the same total mass flow rate $\dot{m}$ has two components, $\dot{m}_1$ and $\dot{m}_2$. These components are, respectively, associated with velocities u_1 and u_2, thrust components $F_1 = \dot{m}_1u_1$ and $F_2 = \dot{m}_2u_2$, and thrust powers $P_1 = F_1^2/2\dot{m}_1$ and $P_2 = F_2^2/2\dot{m}_2$.

The mentioned cases have been constructed so that $\dot{m} = \dot{m}_1 + \dot{m}_2$. Moreover, assume that $F = F_1 + F_2$. In other words assume that the total mass flow rate, the total thrust, and thus the average specific impulse are the same in both cases. Under such conditions, it is straightforward to see that

$$P_1 + P_2 \geq P \quad (1)$$

where the equality sign holds when $u_1 = u_2$. Otherwise, i.e., when u_2/u_1 is different than unity, the total thrust power of Case II is always larger than the thrust power of Case I. The relative thrust power difference is found by simple algebra to be

$$(P_1 + P_2 - P)/P = a_2(1 - a_2)[(u_2/u_1) - 1]^2 \cdot [(1 - a_2) + (a_2u_2/u_1)]^{-2} \quad (2)$$

where a_2 is the mass flow fraction

$$a_2 = \dot{m}_2/\dot{m} = \dot{m}_2/(\dot{m}_1 + \dot{m}_2) \quad (3)$$

associated with velocity u_2 in Case II. It is easily seen in Eq. (2) that the relative power difference is always a positive number, except it is zero, when $u_2 = u_1$. It is also interesting to note that for a given velocity disparity ratio u_2/u_1 the relative power difference given by Eq. (2) has a maximum, when $a_2 = u_1/(u_1 + u_2)$. This maximum is given by

$$[(P_1 + P_2 - P)/P]_{\max} = [(u_2/u_1) - 1]^2/(4u_2/u_1) \quad (4)$$

The aforementioned relations are conveniently illustrated in Fig. 1. Here we plot the relative thrust power difference, designated as relative excess power, vs the mass flow ratio $\dot{m}_2/\dot{m}$, for three different values of the velocity disparity ratio. Note that in all cases of this figure the total thrust, total mass flow rate and the average specific impulse are, by definition, the same and are equal to the corresponding quantities of a flow with no velocity disparities. However, if a fraction, say 10%, of the flow has a velocity disparity ratio equal to 3, 10 or 30, then the excess thrust power would be correspondingly 25%, 200% or 550%, of the thrust power required in the case of a uniform flow.

It is also realized that over and above the spatial and temporal nonuniformities mentioned previously, one must consider similar effects arising from velocity disparities in plasma flows where the propellant is a multicomponent fluid. This is especially true in the acceleration of a partially ionized gas by electromagnetic means. Here the thrust power is originally invested in the ionic species of the flow. Consequently, large velocity disparities between the ions and the neutrals may arise, if the coupling for momentum transfer is weak.

The present work is concerned primarily with the investigation of ion-neutral velocity disparities in plasma flows of propulsion interest.

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Table 1 Experimental conditions regarding the configurations of Fig. 3

	Configuration a	Configuration b
Gas	Ammonia and/or nitrogen	Nitrogen
Flow rate	10–100 mg/sec	100–500 mg/sec
Chamber pressure	2–4 mm Hg	30–100 mm Hg
Environmental pressure	10–100 microns Hg	40–80 microns Hg
B field	<300 gauss	zero
MPD current	1000–2000 amp	1000–2000 amps
Power input	50–120 kw	40–100 kw
Additives (in very small fractions)	Helium, argon, neon and krypton	Helium, argon, neon and krypton

II. Background

One wishes to obtain a certain thrust from an MPD thrustor, ideally and most efficiently, by accelerating all the components of the propellant to a single axial velocity, with negligible radial gradients. In practice however, the observed picture is distressingly different. This is especially so in the most usual, steady-state MPD thrustor, operating at currents in the range of a few hundred amperes, external magnetic fields up to a few thousand gauss, and usually, with ammonia propellant at rates of the order of 10 mg/sec.

Under such conditions, several investigators have observed recently and reported strong departures from a uniform axial flow velocity. These findings may be summarized as follows:

a) The ionic species of the propellant have substantial rotational velocities.[1,2] This has been most readily demonstrated by the familiar slanting of ionic spectral lines, Fig. 2a, observed when the MPD flow is side-viewed. In this case, as would be expected from a rotating propellant, the part of the spectral lines originating from above the center line of the flow is Doppler shifted in one direction, whereas the opposite is true at the other end of the flow diameter. Generally, the mentioned rotational velocities increase with increasing MPD current and magnetic field; compare Fig. 2a with Fig. 2b, and they reach values comparable to the axial velocity of the propellant. Similarly, substantial rotational velocities have been measured and reported[3] for ionized argon in an argon MPD flow.

b) The presence of substantial rotational velocities of the ionic species, unjustified on gasdynamic grounds, leaves no doubt about the combined action of the electric and magnetic fields on the ionized part of the propellant. In this case, radial currents and axial B fields, and/or conversely, may combine to produce azimuthal acceleration. However, it is quite important to note[1,2] that no evidence has been found that the neutral species in the same MPD flow have detectable rotational velocities. The familiar slanting effect of the ionic spectral lines is absent in the case of neutral lines. This is a definite indication of poor ion-neutral coupling, for momentum transfer.

c) More important is the situation of the observed axial velocities, under the mentioned experimental conditions. Here again, very substantial disparities between ionic and neutral velocities have been reported.[1,2,4] Generally, the neutral species have detectable velocities with values justified on gasdynamic grounds (thermal expansion), whereas the ions have velocities several times higher, justifiable only by virtue of electromagnetic acceleration. Other investigators[5] have also observed relatively modest ion-neutral velocity disparities. However, the same investigators have reported[6] a rather complicated dependence of velocity disparities on the environmental pressure of an MPD flow.

d) Further evidence, regarding the weak coupling of the propellant species, was found[1] in terms of the relatively strong dependence of the particle velocities (both ionic and neutral), on the particle mass. In these experiments, small amounts of various noble gases were added in the regular propellant. It was found that the particle velocities depend, not only on the ionic or neutral state of a particle, but also on its mass. Clearly, the heaviest particles were found to have the lowest velocities. Such a situation could not arise, if the particles were strongly coupled for momentum transfer.

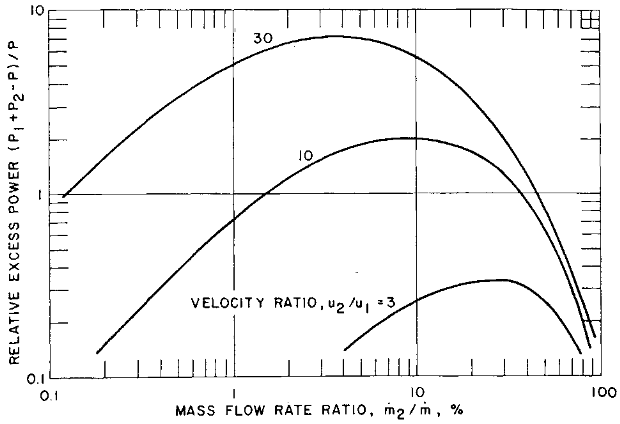

Fig. 1 Relative excess power caused by nonuniformities vs the mass flow fraction for three different ratios of velocity disparities.

In conclusion, the aforementioned findings seem to indicate that the velocity field in typical, steady MPD flows is rather complicated and very nonuniform. Specifically, the ionic species of the propellant appear to gain not only axial but also rotational kinetic energy, from electromagnetic forces. The neutrals, on the other hand, have much lower velocities, mostly of gasdynamic origin. Moreover, for all practical purposes, they are weakly coupled to the ions, and they share little of the momentum gained by the ions electromagnetically.

III. Experimental

Beyond the background outlined in the last section, further experimental work is reported here, regarding the determination of ion-neutral velocity disparities in MPD flows. In these more recent experiments two MPD configurations were employed, as shown in Figs. 3a and 3b. The corresponding ranges of experimental conditions are summarized in Table 1. A comparison of the present arrangements and conditions with typical[1-6] MPD conditions shows that, basically in the present work, the mass flow rate has been increased by one to two orders of magnitude, the current has been increased by a factor as large as 5, the externally applied B field has been made either very small or zero, and the input power has been increased by roughly an order of magnitude.

These experimental conditions are by no means suggestive of more attractive MPD conditions, for practical applications.

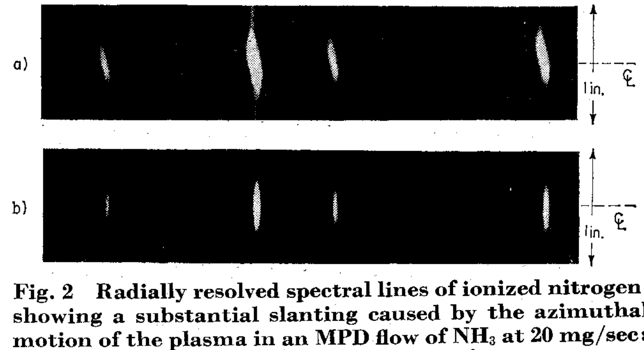

Fig. 2 Radially resolved spectral lines of ionized nitrogen, showing a substantial slanting caused by the azimuthal motion of the plasma in an MPD flow of NH_3 at 20 mg/sec: strongest line at 5680 Å; total span ~27 Å. Case a): 800 amp 3000 gauss; case b): 300 amp 400 gauss.

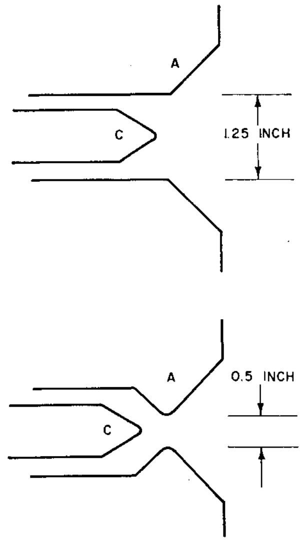

Fig. 3 Two MPD configurations employed in the experiments; A: anode, C: cathode. Upper part, configuration a; lower part, configuration b.

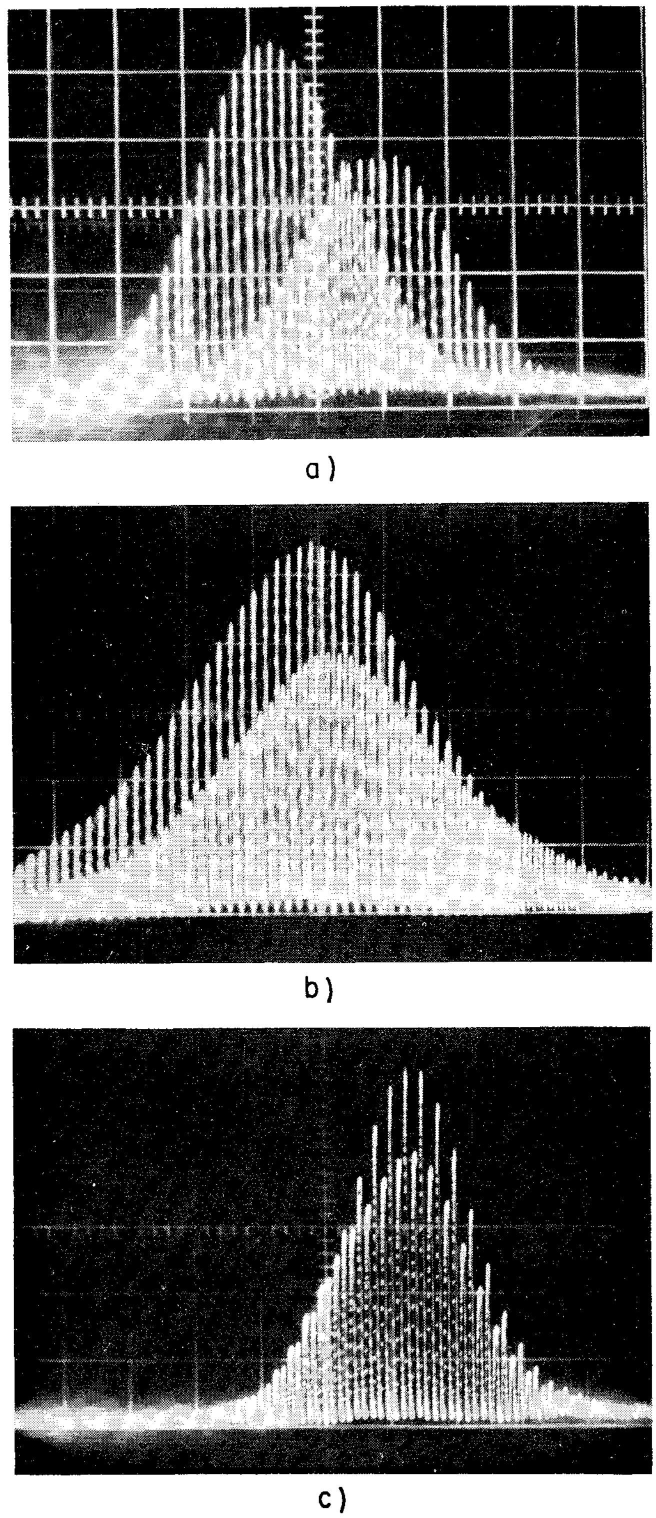

Fig. 5 Examples of spectral line shifts; total scan in each case: 1.25 Å: a) ionized nitrogen in an MPD flow, b) neutral hydrogen in an MPD flow, c) mercury from a stationary Geissler tube.

They are simply employed here for the investigation of the problem of ion-neutral coupling in MPD flows. The following motivation is relevant. First, the drastic increase of the mass flow rate is intuitively clear. As will be explained later, the absence of strong ion-neutral coupling in typical MPD devices can be assigned to insufficient particle densities in the acceleration region. Obviously then, a large increase of the flow rate, without correspondingly large increases in flow cross section and axial velocity, could help in strengthening the coupling in question. However, it is also understood that a drastic increase of the particle density does not necessarily promote good ion-neutral coupling, unless accompanied by adequate ionization.

Thus, intuitively speaking, we would expect a stronger ion-neutral coupling when an increased mass flow rate is accompanied by an increase of the power input. The power increase, in the present experiments, was obtained mostly by an increase of the MPD current. This was not only necessary, but also very desirable. At high currents, the self-induced magnetic field (~1000 gauss at 2000 amp), may be as adequate as typical externally applied B fields. At the same time, the azimuthal self-field is more preferable, since, unlike an externally applied B field, it does not promote the familiar spoke formation and rotation.[7] The formation and rotation of spokes in MPD arcs is undesirable for several reasons. Most important here is that a spoke may engage only a small fraction of the propellant in the electromagnetic interaction region. This then concludes the motivation for conducting the present MPD experiments, under the conditions of Table 1.

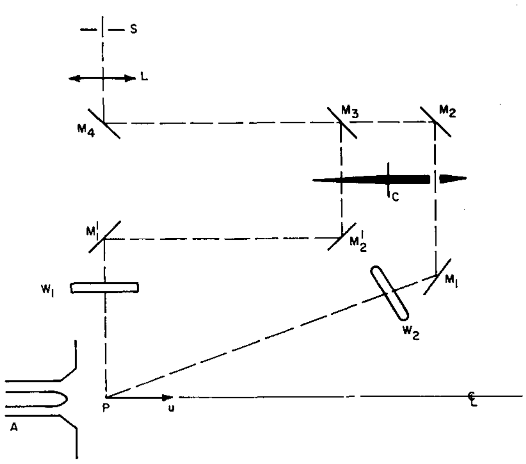

Fig. 4 Experimental arrangement for the determination of Doppler shifts caused by axial motion, A: accelerator, W: environmental tank windows, M: mirrors, L: lens, S: spectrograph entrance slit, C: chopper.

Initially, experiments were performed with the configuration a of Fig. 3 and Table 1. As will be seen in the detailed results of the next section, no evidence of strong ion-neutral coupling was found, even at flow rates as high as 100 mg/sec. The mass flow rate could not be increased well above this level, in configuration a, without the onset of gross instabilities in the MPD accelerator. In fact, even at lower flow rates, a small external B field (<300 gauss), had to be applied in order to prevent gross instabilities. Configuration b of Fig. 3 and Table 1 was then adopted as a desirable alternative, for operating at higher flow rates. It was known from previous experiments,[8] that this configuration could operate without major problems at high flow rates, high currents and zero external B field. It is also important to note that, in configuration b, advantage may be taken not only of higher flow rates, but also of higher values of the mass flux through the energy addition region. This is evident by inspecting the two geometries of Fig. 3.

The velocities and velocity disparities under consideration here were determined spectroscopically by measurements of the Doppler shifts of selected spectral lines, for several species both ionic and neutral. Also, as shown in Table 1, several additives in small‡ fractions were used in order to study the dependence of velocities and velocity disparities on the particle mass.

The experimental arrangement for the Doppler shift measurements has been described in detail previously.[1] Here, the essential aspects are summarized by reference to schematic presentation of Fig. 4.

The chopper improves substantially the sensitivity and reliability of this arrangement. It is used in such a way that either of the optical paths may pass while the other is blocked. This is done alternatively and repetitively, many times, during any single scan of a spectral line. As a result of this, the radiation from both optical paths under comparison has one and the same wavelength scale, whereas individual scales can be conveniently maintained for the relative intensities.

Examples of Doppler shifts, measured photoelectrically and recorded on polaroid shots, are given in Figs. 5a and 5b. The experimental arrangement is frequently tested against shifts of non-Doppler origin, by the use of a stationary source of radiation. The results of such a test are given in Fig. 5c where a zero shift is observed, as would be expected from a stationary source. Also, as a test, the side path of Fig. 4, which provides the unshifted wavelength, has been replaced in several cases by a path originating from a stationary source, independent of the MPD flow. Such test measurements differ from ordinary measurements by less than $\pm 10\%$.

Strictly speaking, the Doppler shift measurements reported here are not spatially resolved. Most of them refer to the core, ($\frac{1}{2}$–$\frac{3}{4}$ in.) of the MPD flow and have been obtained with line intensities and shifts integrated along lines of sight, passing through the centerline of the flow. However, the Doppler shift contributions, from the thicker and much slower gas layers surrounding the core, were effectively rejected. This was done by carefully selecting spectral lines which have strong intensities in the core, but much weaker intensities outside the core. Radial profile estimates of the axial velocities were obtained from auxiliary measurements, in which the Doppler shifts were observed along lines of sight off the center of the flow.

The most frequently used spectral lines, which qualified for the measurements reported here, are identified in Table 2. It is noted that for one and the same particle, several wavelengths could be used. This was done frequently, as a test, in the determination of one and the same velocity, by using several different wavelengths. It is also noted that no spectral lines are available for the ions of hydrogen, helium and neon. In the case of hydrogen, naturally, no ionic lines exist, whereas the ionic spectral lines of helium and neon could not be excited, under the present conditions, because of their relatively high excitation potentials.

Table 2 Most frequently used spectral lines

Species	Wavelength, Å
Hydrogen atom	6562
	4861
	4340
Helium atom	6678
	5875
Nitrogen atom	7469
	7443
Nitrogen ion	5680
	5005
	3995
Neon atom	6402
	5852
Argon atom	6965
	7067
Argon ion	4806
Krypton ion	4659

‡ Generally a few percent or less of the main propellant flow, which was either ammonia or nitrogen.

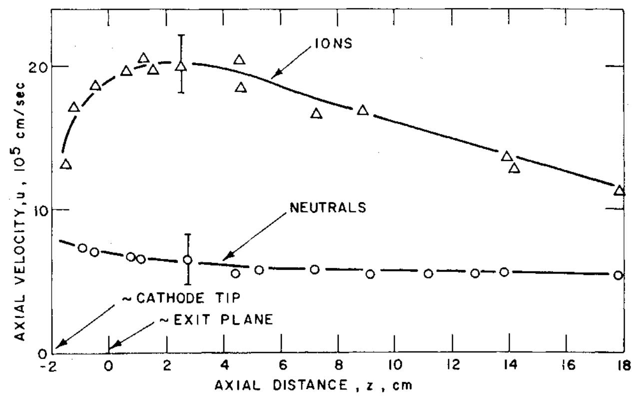

Fig. 6 Weak coupling: data regarding configuration of Fig. 3a; ammonia flow at 100 mg/sec, 1500 amp, with B < 300 gauss.

IV. Results

Briefly and essentially, the results of this work show no evidence of strong ion-neutral coupling in the case of configuration a, (Fig. 3 and Table 1). Moreover, ample evidence of strong coupling is found in the experiments with configuration b, (Fig. 3 and Table 1).

Weak Ion-Neutral Coupling

Typical results regarding the weak coupling cases are illustrated in Figs. 6 and 7 and in Table 3. The velocities of Fig. 6 refer to the core ($\frac{1}{2}$–$\frac{3}{4}$ in.) of the flow. Ionic veloci-

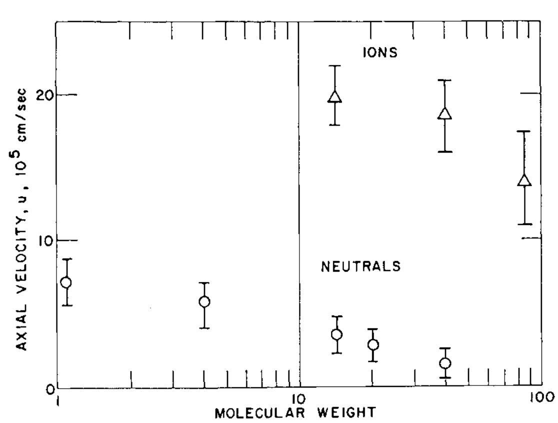

Fig. 7 Weak coupling: data regarding configuration of Fig. 3a; ammonia flow at 100 mg/sec, 1500 amp, with B < 300 gauss.

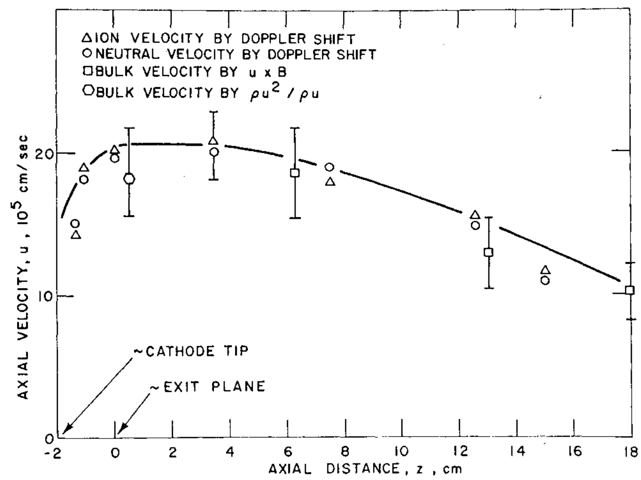

Fig. 8 Strong coupling: data regarding configuration of Fig. 3b; nitrogen flow at 400 mg/sec, 1500 amp and B = 0.

ties are based mostly on nitrogen ions which occur naturally in an ammonia flow, while atomic hydrogen, also naturally occurring in the same flow, has provided the neutral velocities. Each point represents a straight average of many measurements, at least four or more, and the bars indicate typically the spread of these measurements. It is clearly seen that the ions are accelerated from relatively low velocities, in the vicinity of the cathode,§ to velocities just over 2×10^6 cm/sec. These ionic velocities decay rather slowly as the ions travel axial distances of many flow diameters downstream of the accelerator exit. At the same time, the much slower neutrals show no evidence either of original acceleration or of momentum gain at the expense of ionic momentum further downstream. Apparently, the slow decay of ionic velocities at large axial distances reflects the core decay of the MPD flow by mixing. In this case the ionic momentum loss is transferred to much larger amounts of gas. The neutrals might have small velocity gains that are hardly detectable. Regardless of these considerations, however, the very large disparities between ionic and neutral velocities indicate clearly the absence of good coupling in this case.

As mentioned, the ionic velocities of Fig. 6 refer to nitrogen ions, whereas hydrogen atoms have provided the neutral velocities. However, the velocity disparities between ions and neutrals, of the same particle mass, are more serious than those reflected in Fig. 6. This may be seen in the data of Fig. 7, where ionic and neutral velocities are given vs the particle mass. Hydrogen, helium, nitrogen, neon, argon and krypton species are found at molecular weights equal to 1, 4, 14, 20, 40 and 80, respectively. In these experiments, small additive fractions of noble gases have been used in the main, ammonia, propellant. The velocities reported here refer as always to the flow core and have been obtained at an axial distance, ~3 cm, corresponding to the appearance of the broad maximum in the ionic velocities of Fig. 6. The ion-neutral velocity disparities may be appreciated by reference to Fig. 7, at a molecular weight equal to 14, nitrogen, and at 40, argon. The data of Fig. 7 provide also evidence showing a very substantial dependence of the particle velocity on the particle mass. This effect is especially strong for the neutrals and could not arise if a flow velocity existed, common for all particles.

The data of Figs. 6 and 7 do not change substantially for conditions other than those specified in the aforementioned figures, but within the ranges defined in Table 1 for configuration a. For example, refer to Table 3, which presents the dependence on the MPD current. Similarly, a variation of the mass flow from 10 to 100 mg/sec does not alter significantly the presented data. Finally, as mentioned before, the external application of strong B fields introduces additional evidence of weak ion-neutral coupling, since in this case the ions acquire substantial rotational velocities, whereas the neutrals do not.

§ Axial distances closer to the cathode could not be covered because of interference from the continuum radiation of the hot cathode.

Before concluding this topic, it is worth noting that the neutral velocities of Table 3 are justifiable by gasdynamic considerations, while the ionic velocities are several times higher than average gasdynamic estimates. Such estimates may be obtained from the plenum pressures tabulated in Table 3 and from the ~8 cm² cross section of the MPD throat in Fig. 3a. For simplicity, a thrust coefficient equal to unity may be assumed. Then at 5-mm Hg plenum pressure and at 100 mg/sec flow rate, we find an average gasdynamic velocity of 5.2×10^5 cm/sec for the data of Table 3.

Strong Ion-Neutral Coupling

Typical results illustrating the strong coupling cases are given in Figs. 8 and 9 and in Table 4, which all refer to configuration b of Fig. 3 and Table 1. Again all velocities refer to the flow core unless otherwise specified.

The ion velocities of Fig. 8 are determined by Doppler shift measurements, and they are averages of many measurements made for nitrogen, argon and krypton ions. Similarly, the neutral velocities are averages obtained from many measurements with helium, nitrogen, neon and argon neutrals. This averaging process is allowed here, both for ions and for neutrals, because the particle velocities are virtually independent of the particle mass, as may be seen in the data of Fig. 9. Note that nitrogen is the main propellant and that all the noble gases are small fraction additives. According to this evidence, all particles have a common velocity regardless of their ionic or neutral state and regardless of their mass. This common velocity increases rapidly in the acceleration region, close to the electrodes. It reaches values just over 2×10^6 cm/sec, and it decays slowly at large axial distances where copious mixing with the environmental gas may be taking place.

Two additional and independent velocity determinations have been superimposed on the ionic and neutral velocities of Fig. 8. They are the bulk plasma velocity as determined by a $u \times B$ probe,[8] and the bulk gasdynamic velocity, determined by combined momentum flux and mass flux probing.[8,9] In both cases, sufficient radial resolution was available for an average velocity determination in the flow core ($\frac{1}{2}$–$\frac{3}{4}$ in.). It is clear that all particle and bulk velocities plotted in Fig. 8 are basically in agreement, as would be expected in a case of strong coupling between any two components of a fluid.

The discussed evidence of strong ion-neutral coupling is not detectably affected by a variation of the experimental conditions, as defined in Table 1, for configuration b. All velocities (neutral, ionic, and bulk) increase appreciably when the MPD current increases and/or when the flow rate decreases. However, a common velocity remains always in evidence, as may be seen in the data of Table 4.

Table 3 Weak ion-neutral coupling; dependence of ion and neutral velocities, MPD voltage and plenum pressure on the accelerator current, at $\dot{m}$ = 100 mg/sec, ammonia, and B <300 Gauss

I Amperes	V Volts	p_0 mm Hg	Neutrals $u^0\ 10^5$ cm/sec	Ions $u^+\ 10_6$ cm/sec
1000	50	5	5.9	20
1500	55	4	5.6	20
2000	60	4	5.5	19.5

V. Analytical Considerations

It is considered helpful here to formulate a simple analysis for the interpretation and evaluation of the results regarding the ion-neutral coupling under the present experimental conditions as well as under more typical MPD conditions.

The present experimental data, Figs. 6 and 8, show clearly that most of the acceleration (either for the ions or both for the ions and for the neutrals), occurs mainly between the cathode tip and the accelerator exit. This is not unexpected here, since it is known[9] that the axial current density declines rapidly and becomes negligible outside the accelerator exit. It must also be taken into account here that the magnetic field is mostly self-magnetic. Thus very little acceleration would be expected downstream of the accelerator exit. In other studies[2,6,1] under most typical MPD conditions (strong external B fields, much lower flow rates), substantial ionic acceleration takes place in the region immediately downstream of the accelerator exit. This is also understood, since, under such conditions, substantial current densities are detected[10] downstream of the accelerator exit, where also substantial applied B fields exist. In either case, the axially identified acceleration region, has been also identified radially as a relatively small tube, confined in the core of the MPD flow. Such observations have been provided by current density probing,[10] and, in the present work, also by auxiliary Doppler shift measurements.

The question of ion-neutral coupling is obviously meaningless when a totally ionized gas is considered. However, here we are interested in partially ionized flows, where ions are originally accelerated, and where any serious disparities between ion and neutral velocities are undesirable. Therefore, it is important to formulate a criterion, relevant to the strength of the coupling for momentum transfer from the ions to the neutrals.

Define, first, a radius R within which the plasma is confined and accelerated. This radius will be identified more precisely in the following analysis. However, it is already clear that R is not necessarily related to any hardware radius of the accelerator. This is especially so in the cases where the axial acceleration has been observed to take place, either downstream of the hardware exit of the accelerator, or at most within a flow core, with a radius small compared to a rapidly diverging hardware nozzle.

With the radius R so defined we suggest

$$\lambda_0 \ll R \tag{5}$$

as a criterion for strong ion-neutral coupling, where

$$\lambda_0 = (1/n_+ Q_{ex}) \tag{6}$$

is the mean free path of a neutral between collisions with ions, for momentum exchange. In Eq. (6), we consider n_+, the ion density, because only the ions are originally subject to confinement and acceleration forces. The momentum exchange cross section Q_{ex} appearing in Eq. (6) has values[11] generally between 5×10^{-16} and 5×10^{-15} cm^2, depending on the colliding particles and their relative velocity. The highest values are applicable in cases of resonant charge exchange, in collisions between ions and neutrals of the same atom.

The ion density is related to the total particle density in the plasma n by the relation

$$n_+ = an \tag{7}$$

where a is the degree of ionization. A combination of Eqs. (5–7) gives

$$n\,R \gg (1/aQ_{ex}) \tag{8}$$

We suggest that when this criterion is satisfied, a strong coupling will be established between the ions and neutrals in the important region of the accelerator. Otherwise, the neutrals would leak out, while sharing very little of the momentum gained by the ions, electromagnetically. An equally simple criterion could be applied for the axial acceleration of the neutrals. In this case, the radius R should be replaced by the axial dimension L of the acceleration region. However, from experimental studies we know that $L > R$ in most cases of interest. Thus if criterion Eq. (8) is satisfied, then the condition for good ion-neutral coupling, in the axial direction, will also be satisfied.

For the evaluation of criterion Eq. (8), we clearly need the particle density and the confinement radius of the plasma in the acceleration region. The following analysis appears to be relevant. First, it is noted that, no matter how strong the confining B field is, and how high the plasma conductivity is, there will always be a radial diffusion of the plasma across the B field. This will be also so regardless of the B field origin, external or self-induced. Let u_D be the diffusion velocity of the plasma. Clearly, if the plasma is accelerated over an axial length L, with an average velocity u_z, the confinement radius R will be augmented by

$$\Delta R \simeq u_D\,(L/u_z) \tag{9}$$

In general we wish $\Delta R/R$ to be as small as possible in the confinement and acceleration region of interest. Otherwise, the mass continuity will reduce the particle density, and this is undesirable. Arbitrarily here we set $\Delta R/R \simeq 0.2$, which means an augmentation of the flow cross section by about 50%, along the acceleration channel. Thus, relation (9) becomes

$$u_D \simeq (u_z R/5L) \tag{10}$$

The radial diffusion velocity of the plasma may be related now to other important parameters, as follows. The physical consequence of radial plasma diffusion against an orthogonal B field, (either axial and/or azimuthal), is the generation of

Table 4 Strong ion-neutral coupling; dependence of ion, neutral, and bulk velocities on the accelerator current; also, MPD voltage and plenum pressure, at $\dot{m}$ = 400 mg/sec, nitrogen and $B = 0$

I Amperes	V Volts	p_0 mm Hg	Neutrals u^0 10^5 cm/sec	Ions u^+ 10^5 cm/sec	Bulk $\langle u \rangle$ 10^5 cm/sec
1000	40 ± 5	60 ± 5	16	17	15
1500	43 ± 5	70 ± 5	21	20	19
2000	47 ± 5	85 ± 5	23	23	24

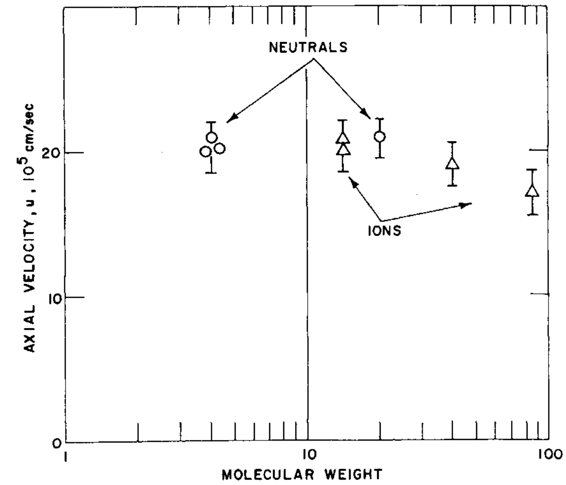

Fig. 9 Strong coupling: data regarding configuration of Fig. 3b; nitrogen flow at 400 mg/sec, 1500 amp and B = 0. All data, at about 3 cm from the accelerator exit.

an induced current density j given by

$$j = \sigma u_D B \tag{11}$$

which is also orthogonal to both u_D and B. The plasma conductivity here, is represented by σ. In turn, the induced current density combines with the B field to provide a radial force jB which opposes and, in the steady state, balances the pressure gradient ∇p in the radial direction. This gradient here may be approximated by p/R, where $p = nkT$ is the average plasma pressure, at an average plasma temperature T, inside R. When all relations, following Eq. (10), are combined, it is found that

$$n/R^2 = (\sigma B^2/kT)(u_z/5L) \tag{12}$$

This relation only is not adequate for the evaluation of criterion in Eq. (8). Obviously a second relation between n and R is needed. Intuitively, we feel that the mass continuity through the acceleration tube must be considered, since the mass flow rate has not appeared yet in the present analysis. For this, we write simply

$$nmu_z \pi R^2 = \dot{m}$$

or

$$nR^2 = (\dot{m}/\pi m u_z) \tag{13}$$

where $\dot{m}$ is the mass flow rate¶ through the region of interest and m is the average particle mass.

The particle density and the confinement radius in the acceleration region may be obtained from Eqs. (12) and (13), in terms of the quantities appearing on the right-hand sides. More important here is the product nR appearing in criterion in Eq. (8). It is found that

$$nR = [(5\pi^3 L)^{-1}(\dot{m}/m)^3(\sigma/kT)(B/u_z)^2]^{1/4} \tag{14}$$

A numerical evaluation of criterion in Eq. (8) is now in order. We shall use Eq. (14), assigning values to the quantities of the right-hand side, as suggested by relevant experimental observations. Typical MPD conditions are considered first. It appears reasonable to take $u_z \simeq 10^6$ cm/sec as an average plasma velocity over an acceleration length $L \simeq 2$ in. A magnetic field of 1000 gauss appears appropriate, and for a flow rate of 10 mg/sec, e.g., of ammonia or nitrogen, we find that $(\dot{m}/m) \simeq 5 \times 10^{20}$, approximately. Finally, with a typical temperature of 10^4 °K and for a conductivity of 10^3 mhos/m, we obtain $nR \simeq 1.6 \times 10^{14}$ particles/cm^2. This results from a combination of $n \simeq 2 \times 10^{14}$ particles/cm^3 with a confinement radius $R \simeq 0.8$ cm, as may be found by using independently Eqs. (12) and (13).

So far, there is nothing incredible in the values estimated for n and R. However, throughout this analysis, the plasma has been treated as a single component fluid, and this may be physically acceptable, only if criterion in Eq. (8) is satisfied. Reference to this relation shows that it is not satisfied, under the aforementioned MPD conditions. The right-hand side of Eq. (8) assumes the lowest value, of about 4×10^{14} cm^{-2}, when a degree of ionization as high** as 50% is combined with the upper limit of the exchange cross-section range, i.e., $Q_{ex} \simeq 5 \times 10^{-15}$ cm^2.

In summary then, typical MPD conditions require $nR \simeq 1.6 \times 10^{14}$ cm^{-2} while the right-hand of criterion in Eq. (8) could not possibly have a value lower than 4×10^{14} cm^{-2}. Thus the criterion in question is not satisfied, and no strong ion-neutral coupling may be expected. Naturally, lower degrees of ionization imply an even weaker coupling. Otherwise, the sensitivity of criterion in Eq. (8) to the particular experimental conditions may be evaluated when the right-hand side of Eq. (14) is inspected. Note that L, σ and T, which anyway cannot be drastically altered in favor of stronger ion neutral coupling, have a rather weak influence ($\frac{1}{4}$ power) on nR. Similarly, B and u_z have a stronger influence, $\frac{1}{2}$ power, but practical consideration would not allow much higher values for B, or much lower values for u_z, than assumed here.

Thus, it appears that $\dot{m}$ as a parameter could affect nR, most significantly. This is so, not only because of the $\frac{3}{4}$ power dependence in Eq. (14), but also because $\dot{m}$ may in principle be varied over a range, much wider than that of any other parameter. A few comments are in order here, regarding the value of $\dot{m}$ which appears in Eq. (14), and is the actual flow rate through the acceleration region. As noted previously in connection with Eq. (13), $\dot{m}$ is not necessarily the metered value of the mass flow rate. This happens because of the actual mass flow rate through the acceleration region may be seriously affected by two opposing influences. One is that a substantial fraction of the metered flow rate may altogether bypass the acceleration region, and the other is an environmental influence, namely, entrainment of gas from the environmental tank. In the concluding remarks, the aforementioned considerations are applied in the evaluation of three situations: a) the evidence of weak coupling[1,2,4] for typical MPD conditions, associated with previous work[1–6]; b) the evidence of weak coupling, associated with the data and conditions of Fig. 6; and c) the evidence of strong coupling associated with the data and conditions of Fig. 8.

VI. Concluding Remarks

a) Experimental work here and elsewhere[1,2,4] shows no evidence of strong ion-neutral coupling in the case of typical MPD conditions, as defined in the discussion following Eq. (14). It is quite possible that, in these situations, a very substantial fraction of the metered flow rate bypasses the acceleration region. If this is a fact, then it is important to ask whether a strong coupling would be expected if all the metered flow rate was passing through the acceleration region. The answer to this question appears to be negative, as shown by the discussion which follows [Eq. (14)]. Regardless of this negative answer, one more outstanding question exists, namely, what determines and influences the fraction of the metered flow rate which is engaged in the acceleration region?

b) The presence of weak coupling, under the conditions of Fig. 6, is experimentally obvious. The question is whether this is analytically expected. The answer here is that the coupling is analytically expected to be weak or at best marginal. This may be seen by repeating the procedure following Eq. (14). In the present case, the flow rate is 100 instead of 10 mg/sec, ammonia, and the external B field is replaced by a comparable self-induced B field. Otherwise, the same general conditions are applicable. When Eq. (14) is used, it is found that $nR \simeq 9 \times 10^{14}$ cm^{-2}, which must be compared with 4×10^{14} cm^{-2} on the right-hand side of criterion in Eq. (8). Thus the situation appears to be at best marginal, even when the full value of the metered flow rate is used in Eq. (14).

c) Finally, the strong coupling of Fig. 8 is both experimentally obvious and easier to justify analytically. Here, not only the flow rate is four times higher than in the last case, but also, the configuration is less favorable to the propellant bypassing the acceleration region. In this case, an application of Eq. (14) yields $nR \simeq 26 \times 10^{14}$ cm^{-2}, which must be compared with 4×10^{14} cm^{-2} on the right-hand side of criterion in Eq. (8).

A few more comments are still necessary regarding the environmental interference known as entrainment. Regarding the aforementioned cases b, and c, it has been determined experimentally[12] that, under the specified experimental conditions, entrainment is fractionally small. Quantitative experiments,[12] with tracer gases in the environmental tank and sample taking from several flow stations, have shown that the fractional amount of the environmental gas in the MPD

¶ Not necessarily the metered flow rate.

** Much higher degrees of ionization would not be relevant in a problem of ion-neutral coupling.

flow, varies between a few and ten %, depending on axial and radial station.

Entrainment data, on a quantitative basis, are not available for the aforementioned a. However, qualitative evidence shows that substantial entrainment could be taking place. Fractionally, it could be well above 10%, especially since the propellant flow rates are lower in this case. Nevertheless, it is important to note that copious entrainment could falsify the experimental evidence, only when such evidence indicates good coupling. If, as the case is here, the experimental evidence indicates poor coupling, in spite of the fact that environmental gas might enter copiously into the acceleration region, then the said coupling could not become any stronger in the absence of entrainment.

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Acceleration Patterns in Quasi-Steady MPD Arcs

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Argon exhaust velocities of the order of 25,000 m/sec are observed in a quasi-steady, self-field MPD accelerator operated at 17,500 amp. The variation of measured exhaust velocity with input mass flow indicates a preferred or "matched" mass flow, at which the observed velocity substantially exceeds both the Alfvén critical speed and that computed from the electromagnetic thrust relation. Although the measured velocity corresponds roughly to the total electrode voltage, an electrostatic ion acceleration mechanism is not supported by detailed maps of the potential contours within the arc chamber. Specifically, it is found that the bulk of the arc voltage gradient, exclusive of the electrode falls, occurs within two diameters of the cathode, and is normal to it. The remainder of the arc chamber, and all of the downstream plasma is nearly equipotential, at a value close to anode sheath potential. Anode fall voltage varies inversely with local current density, implying substantially lower anode losses at higher power arc operation. Bow shocks from small obstacles placed in the exhaust stream indicate supersonic, but not hypersonic flow.

I. Introduction

QUASI-STEADY plasma propulsion involves the operation of self-field, electromagnetic thrusters in repetitive high-power pulses of sufficient duration that steady acceleration processes prevail over most of the pulse, but at repetition rates commensurate with available space power supplies. Principal attractions of this concept are the combination of arc operation in regimes of high thermal efficiency and dynamic stability with tolerable heat transfer and mean power demands, and the opportunity for variable thrust and mean power consumption without compromise in specific impulse or thruster efficiency, both accomplished via simple duty-cycle adjustment. Possible disadvantages concern the more complex power-conditioning and propellant injection implicit in systems of this type, compared to those using completely steady thrusters. Success of the concept thus hinges on coordinated efforts to minimize these system penalties through sophisticated design and technological improvements, and to exploit the advantages of high-power accelerator operation to the extent that the residual system disadvantages are more than counterbalanced. This paper attempts to contribute to the latter task by identifying some of the operating characteristics and interior physical processes of one particular quasi-steady accelerator.

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